# Chapter 24

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## Gauss's Law Applications



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# Lecture 03

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## Applications Of Gauss's Law



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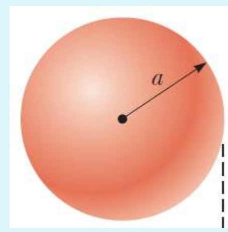
## *Example 1*: Field Due to a Spherically Symmetric Charge Distribution

An insulating solid sphere of radius $(a)$ has a uniform volume charge density $(\rho)$ and carries a total positive charge $(Q)$.

a) Calculate the magnitude of the electric field at a point outside the sphere.

b) Calculate the magnitude of the electric field at a point inside the sphere.



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## Example 1: Field Due to a Spherically Symmetric Charge Distribution

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a) Outside the sphere $(r > a)$

$$E = \frac{1}{4\pi\epsilon_\circ}\frac{Q}{r^2}$$

**E** is Identical to that for a point charge

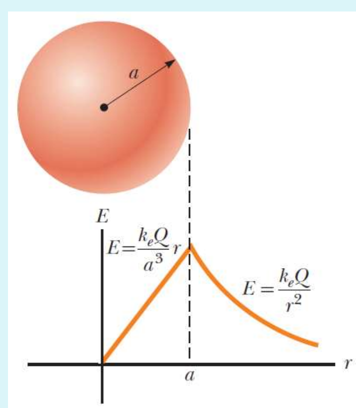
b) Inside the sphere $(r < a)$

$$E = \frac{\rho}{3\epsilon_\circ}r$$

$$E = \frac{1}{4\pi\epsilon_\circ}\frac{Q}{a^3}r$$

**E** varies linearly with $r$

$E \rightarrow 0$ as $r \rightarrow 0$



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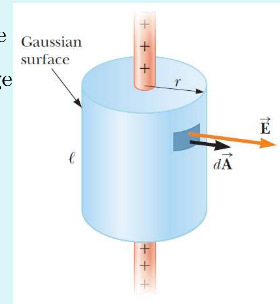
# *Example 2*:
A Cylindrically Symmetric Charge Distribution

Find the electric field a distance $(r)$ from a line of positive charge of infinite length and constant charge per unit length $(\lambda)$.




$$E = \frac{1}{2\pi\epsilon_\circ}\frac{\lambda}{r} = \frac{1}{4\pi\epsilon_\circ}\frac{2\lambda}{r}$$

**E** is inversely proportional to r

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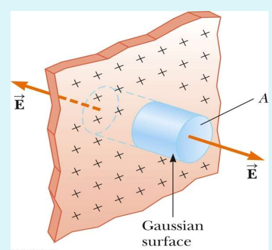
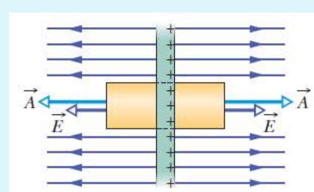
# *Example 3*:
Field Due to a Plane of Charge

Find the electric field due to an infinite plane of positive charge with uniform surface charge density $(\sigma)$.




$$E = \frac{\sigma}{2\epsilon_\circ}$$




Note, this does not depend on $(r)$. Therefore, the field is uniform everywhere.

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# Example 3: Field Due to two Planes of Charge

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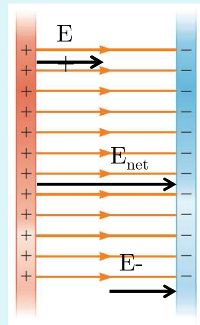
Suppose two infinite planes of charge are parallel to each other, one positively charged and the other negatively charged. The surface charge densities of both planes are of the same magnitude. What does the electric field look like in this situation?

$$E_{net} = E_+ + E_- = \frac{\sigma_+}{2\epsilon_\circ} + \frac{\sigma_-}{2\epsilon_\circ}$$




$$E_{net} = \frac{\sigma}{\epsilon_\circ}$$

Uniform E

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# A Sphere Inside a Spherical Shell

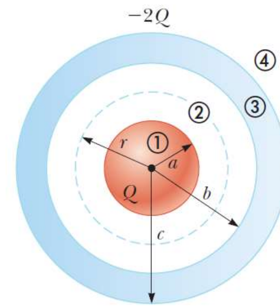
Friday, 29 January, 2021 20:50

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.


- [x] R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- [ ] J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
- [ ] H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
- [ ] H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.


A solid insulating sphere of radius (a) carries a net positive charge (Q) uniformly distributed throughout its volume. A conducting spherical shell of inner radius (b) and outer radius (c) is concentric with the solid sphere and carries a net charge (-2Q).

- find the electric field in the regions labeled: 1, 2, 3 and 4 in the Figure, and
- the charge distribution on the shell when the entire system is in electrostatic equilibrium.




## Solution

**(A) In region 2**, between the surface of the solid sphere and the inner surface of the shell, we construct a spherical Gaussian surface of radius $r$, where $a < r < b$, noting that the charge inside this surface is $+Q$ (the charge on the solid sphere). Because of the spherical symmetry, the electric field lines must be directed radially outward and be constant in magnitude on the Gaussian surface.

The charge on the conducting shell creates zero electric field in the region$r < b$, so the shell has no effect on the field due to the sphere. Therefore, write an expression for the field in region 2 as:

| | |
|---|---|
| $E_2 = k_e \frac{Q}{r^2}$ | for $a < r < b$ |

**In region 1**, because the conducting shell creates zero field inside itself, it also has no effect on the field inside the sphere. Therefore, write an expression for the field as:

| | |
|---|---|
| $E_1 = k_e \frac{Q}{a^3} r$ | for $r < a$ |

**In region 4**, where $r > c$, construct a spherical Gaussian surface; this surface surrounds a total charge $q_{in} = Q + (-2Q) = -Q$. Therefore, model the charge distribution as a sphere with charge $-Q$ and write an expression for the field as:

| | |
|---|---|
| $E_4 = -k_e \frac{Q}{r^2}$ | for $r > c$ |

**In region 3**, the electric field must be zero because the spherical shell is a conductor in equilibrium:

| | |
|---|---|
| $E_3 = 0$ | for $b < r < c$ |

(B) Construct a Gaussian surface of radius $r$, where $b < r < c$, and note that $q_{in}$must be **zero** because $E_3 = 0$. Find the amount of charge $q_{inner}$on the inner surface of the shell:

$$q_{in} = q_{sphere} + q_{inner} \Rightarrow q_{inner} = q_{in} - q_{sphere} = 0 - Q = -Q$$

The charge on the inner surface of the spherical shell must be $-Q$ to cancel the charge $+Q$ on the solid sphere and give zero electric field in the material of the shell. Because the net charge on the shell is $-2Q$, its outer surface must carry a charge $-Q$.